Combinatorics

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Warm Up

Exercise 1. A baby in cloth weighs 10 pounds. The baby weighs 9 pounds more than the cloth. How much does the baby weigh?

Exercise 2. A book costs $5 plus half of its price. How much is the book?

Problem Set

Exercise 3. Massachusetts license plates consist of 4 digits followed by 2 letters. What is the maximum number of license plates that Massachusetts can issue?

Exercise 4. What do you think is the largest number of characters in the Morse code of an English letter? How about a Russian letter? Why?

Exercise 5. What are the odds of winning the jackpot in the Mega Millions game, where you have to match 5 different white balls which are numbered from 1 to 56, and one yellow ball, numbered from 1 to 46? What are your odds of winning exactly $10 dollars, that is to match 2 white balls and 1 yellow ball?

Exercise 6. There are 12 apples and 10 oranges on a tray. Mike takes either an apple or an orange. After that Tom takes an apple and an orange. In which case does Tom have more freedom of choice: if Mike had taken an apple or if he had taken an orange?

Exercise 7. In how many ways can you put a white checker and a black checker on a checkerboard so that the white checker can capture the black checker? In how many ways can you put these two checkers on a checkerboard so that they can capture each other?

Exercise 8. The class 8G has 45 students, 25 of which are boys. 30 students from 8G take Olympiad training; 16 of them are boys. 28 students from 8G play in a band and 18 of those are boys and 17 of those take Olympiad training. 15 boys from 8G take Olympiad training and play in the band at the same time. What is wrong with these numbers?

Exercise 9. Find the largest number that is divisible by 36 and contains every digit from 0 to 9 exactly once.

Exercise 10. Prove that a number written with 6 digits that are all the same is divisible by 3, 7, 11, 13 and 37.

Exercise 11. Mike changed all the same digits in a multiplication example with the same letters, and different digits with different letters. He got: AB * CD = EEFF. Prove, that he made a mistake.

Exercise 12. Do there exist two consecutive natural numbers, such that the sum of the digits for each one is divisible by 11.

Exercise 13. In the line to the movie theater there are $m + k$ people. Out of these people, m people have only $5 bills and k people have only $10 bills. The ticket costs $5, and at the beginning, the cashier doesn't have any change. Also, the cashier only accepts cash. In how many different ways can these people form a line so that there is no problem with change.