Topology and Review

Tanya Khovanova

Department of Mathematics, MIT

May 29, 2008

Warm Up

Exercise 1. Those of you who know anything about Catholicism, do you happen to know if the Catholic Church allows a man to marry his widow's sister?

Exercise 2. A certain street contains 100 buildings. A sign-maker is called to number the houses from 1 to 100. He has to order numerals to do the job. Without using pencil and paper, can you figure out in your head how many 9's he will need?

Problem Set

Exercise 3. Peter has to transfer a head of cabbage, a goat and a wolf from one bank of a river to the other bank. But there is only one seat available on his boat. Furthermore, if the goat and the cabbage stay together as Peter is leaving on his boat, the goat will eat the cabbage. And if the wolf and the goat stay together as Peter is leaving, the wolf will eat the goat.

Exercise 4. Little Red Riding Hood who lives in Belmont went to visit her grandmother in Waltham. She departed at dawn and walked with a constant speed. Her grandmother's neighbor Nif-Nif decided to visit his brother Naf-Naf who is by accident a neighbor of Little Red Riding Hood. Nif-Nif used the same road as Little Red Riding Hood and he also walked with a constant speed. At noon they both passed a sleeping wolf at the side of the road.

They greeted each other, ignored the wolf and continued on their ways. At 4PM Little Red Riding Hood reached her grandmother and at 9PM Nif-Nif arrived at his brother's place. At what time was dawn?

Exercise 5. Parents of two kids, Mike and Tom, want to hire Tanya as a private tutor. Tanya gave the parents an old AMC 10 test and asked them to give it as a practice test to their kids. Mike got 25% of the questions. Tom's parents removed the multiple choice answers from the test and gave Tom just the questions. Tom got 20% of the questions. Tanya had time for only one kid. Whom did she prefer, Mike or Tom? How do you know that this problem is a fictional story?

Exercise 6. There are 5 heads and 14 legs in a family. This family has only dogs as pets. How many dogs are there in this family?

Exercise 7. Prove that $11^{10} - 1$ is divisible by 100.

Exercise 8. A hunter gets out of his cabin one morning to hunt bears. He travels 2 miles south; turns eastward and travels 2 miles and finally turns north and travels for another 2 miles, at the end of which he is back outside his cabin. Now he sees a bear standing outside his kitchen window and eyeing the pie the hunter had left to cool. What is the color of the bear?

Exercise 9. Test for divisibility by 17. Subtract five times the last digit from the remaining leading truncated number. If the result is divisible by 17, then so was the first number. Apply this rule over and over again as necessary. Prove that this test works.

Exercise 10. Mike has 7 different magic cards for exchange and Tom has 9. In how many ways can they exchange one card for another? What if they exchange two cards for two cards?

Exercise 11. How many different words can you get permuting the letters in the word "tanya"? What about "mathematics"?