

Fibonacci Numbers

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Class Discussion

Fibonacci numbers. Lucas numbers. Formula for Fibonacci numbers.

Warm Up

Exercise 1. What are the indices of even Fibonacci numbers? What are the indices of the Fibonacci numbers divisible by 3?

Exercise 2. Don't read this sentence.

Problem Set

Exercise 3. Can you continue the Fibonacci sequence to the negative indices?

Exercise 4. Prove that $\sum_{i=0}^n F_i = F_{n+2} - 1$.

Exercise 5. Prove that $F_{n+1}^2 = F_n F_{n+2} + (-1)^n$.

Exercise 6. Prove that $n^3 + 5n$ is divisible by 6, for any integer n .

Exercise 7. Prove that $1 + 3 + 5 + \dots + (2n - 1) = n^2$.

Exercise 8. Prove that $1 \cdot 1! + 2 \cdot 2! + \dots + n \cdot n! = (n + 1)! - 1$.

Exercise 9. How many binary words of length 11 are there such that every digit appears only an odd number of times in a row?

Exercise 10. Find all prime numbers p such that $p + 10$ and $p + 14$ are also prime.

Exercise 11. Prove that the product of four consecutive integers plus 1 is a square.

Exercise 12. All natural numbers are written one after the other, starting with 1. What digit occupies 206,788th place?