# Fibonacci Numbers 

Tanya Khovanova

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## Class Discussion

Fibonacci numbers. Lucas numbers. Formula for Fibonacci numbers.

## Warm Up

Exercise 1. What are the indices of even Fibonacci numbers? What are the indices of the Fibonacci numbers divisible by 3?

Exercise 2. Don't read this sentence.

## Problem Set

Exercise 3. Can you continue the Fibonacci sequence to the negative indices?

Exercise 4. Prove that $\sum_{i=0}^{n} F_{i}=F_{n+2}-1$.
Exercise 5. Prove that $F_{n+1}^{2}=F_{n} F_{n+2}+(-1)^{n}$.
Exercise 6. Prove that $n^{3}+5 n$ is divisible by 6 , for any integer $n$.
Exercise 7. Prove that $1+3+5+\ldots+(2 n-1)=n^{2}$.
Exercise 8. Prove that $1 \cdot 1!+2 \cdot 2!+\ldots+n \cdot n!=(n+1)!-1$.
Exercise 9. How many binary words of length 11 are there such that every digit appears only an odd number of times in a row?

Exercise 10. Find all prime numbers $p$ such that $p+10$ and $p+14$ are also prime.

Exercise 11. Prove that the product of four consecutive integers plus 1 is a square.

Exercise 12. All natural numbers are written one after the other, starting with 1. What digit occupies 206,788th place?

