# Chinese Remainder Theorem 

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## Class Discussion

Euclid's algorithm. Chinese remainder theorem.

## Warm Up

Exercise 1. 1964 MAML. The tens digit of a certain three digit number is equal to the sum of the hundreds digits and units digit. The number is always divisible by ...?
Exercise 2. 1964 MAML. If the geometric mean between two numbers, $p$ and $q$, is 3 and the sum of their squares is 8 , then what is $(p-q)^{2}$ ?
Exercise 3. 1964 MAML. The difference of the squares of two odd numbers is always divisible by a) 3 , b) 5 , c) 6 , d) 7 , e) 8 ?

## Chinese Remainder Theorem

Exercise 4. What can you say if the number of divisors of a number including 1 and itself is 2 ? 3?

Exercise 5. 2009 HMNT. Find the last two digits of $1032^{1032}$. Express your answer as a two-digit number.
Exercise 6. The product of two numbers is 1000. Each of the numbers is not divisible by 10. Find their sum.

Exercise 7. Peter prepared some lollipops for his party. If he divides them evenly for five people, then two pieces will be left over. If he divides them for four people one lollipop will be left. But he can divide them evenly for three people. What the smallest number of lollipops Peter could have had?

Exercise 8. Find the smallest natural number that have remainders 1, 2, 4, 6 when divided by $2,3,5,7$ correspondingly.

Exercise 9. Find remainders:

1. $19^{10}$ modulo 66
2. $19^{14}$ modulo 70
3. $17^{9}$ modulo 48
4. $14^{14^{14}}$ modulo 100

Exercise 10. Replace stars with digits so that $454^{* *}$ is divisible by 2,7 and 9.

## Competition Practice

Exercise 11. 1964 MAML. If the discriminant of $p x^{2}+2 q x+r=0$, then

1. $p, q$ and $r$ form an arithmetic progression
2. $p, q$ and $r$ are equal
3. $p, q$ and $r$ form a geometric progression
4. $p, q$ and $r$ are all negative
5. none of the above.

Exercise 12. 2009 HMNT. A computer program is a function that takes in 4 bits, where each bit is either a 0 or a 1, and outputs TRUE or FALSE. How many computer programs are there?

Exercise 13. Find all possible integers $N$ that are products of distinct primes, such that $N$ is divisible by every prime divisor reduced by one.

