

Scaling Arguments

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The following statement is false. The preceding statement is true.

Class Discussion

King-Kong. Alice in Wonderland. Bone cross-section, wing size, food.

Warm Up

Exercise 1. Every camper gets their own bowl of soup. Every two campers get one bowl of spaghetti to share. Every three campers get one bowl of salad to share. All campers are required to have their own helping of salad, spaghetti, and soup. How many campers were there if the cook had to prepare 55 bowls of food?

Exercise 2. 2004 AMC 8. At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?

Exercise 3. 2004 AMC 8. Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

Scaling

Exercise 4. Assume a dog's brain is twice as great in diameter as a cat's, but each animal's brain cells are the same size and their brains are the same shape. How many times more brain cells does a dog have than a cat?

Exercise 5. Estimate the number of M&M's in a one gallon container.

Exercise 6. Estimate the number of gallons of water in all oceans.

Exercise 7. Suppose you found Yettie's tracks in Himalayas. The length of Yettie's foot is two feet. Assuming that Yettie should have the same pressure on the ground as humans, so it doesn't get stuck in the mud, how much taller is Yettie than an average human.

Exercise 8. Why would all creatures jump to approximately the same height? You are allowed to find the answer on Internet.

Competition Practice

Exercise 9. 2002 HMMT. Find the greatest common divisor of the numbers $2002 + 2$, $2002^2 + 2$, $2002^3 + 2$, \dots

Exercise 10. 2010 HMNT. What is the remainder when $(1 + x)^{2010}$ is divided by $1 + x + x^2$?

Challenge Problems

Exercise 11. Pooh went from his home to the Piglet's house for a sleepover. He left his home Friday at 6 pm and arrived at the Piglet's house at 8 pm. On Saturday, he left the Piglet's house at 6 pm, walked along the same path, and arrived home at 8 pm. Prove that there exists a point on his route such that he was at that point on Saturday at exactly the same time as on Friday.

Exercise 12. Estimate your weight on a planet with the same density as Earth and which diameter is twice smaller than the Earth's diameter.

Exercise 13. A king has four hats: three blue, and one red. He decides to test his three wise-men. He will put a hat on each man, so that they can see each other, but not their own hat. No communication between the men is allowed. If any wise-man can figure out and say (out loud) to the king what colour hat he has on his head all three go free. What strategy the wise-men should use?