Recursions

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Class Discussion

Recursions. Characteristic polynomials.

Fibonacci numbers: $F_0 = 0$, $F_1 = 1$, $F_{n+1} = F_n + F_{n-1}$. $F_n = (\varphi^n - (-\varphi)^{-n})\sqrt{5} = \dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$.

Lucas numbers: $L_0 = 2$, $L_1 = 1$, $L_{n+1} = L_n + L_{n-1}$.

Warm-Up

Exercise 1. What is the area of a triangle that has sides of 14, 11 and 25 meters long?

Recursions

Exercise 2. Find the explicit formula for the Lucas numbers.

Exercise 3. Prove that $L_n = F_{n-1} + F_{n+1}$.

Exercise 4. Simplify: $1/2! + 2/3! + \ldots + 2000/2001!$.

Exercise 5. Numbers a_0, a_1, ..., a_n, ... are defined as follows: $a_0 = 2$, $a_1 = 3$, and $a_{n+1} = 3a_n - 2a_{n-1}$, for $n > 0$. Find the formula for a_n.

Exercise 6. The Martian alphabet consists of 6 letters that were Morse coded as: $\cdot$, -, $\cdot\,\cdot$, - -, $\cdot$ -, - $\cdot$. Translating the text, the operator forgot to put extra pauses between the letters. The result was a stream of dots and dashes consisting of 12 characters. In how many ways can you read the translated word?

Competition Practice

Exercise 7. MAML 2002. In how many combinations can 5 dollars be paid in dimes and/or quarters?

Exercise 8. MAML 2002. Zoltag the Mind Reader instructs an audience member to write down a four-digit number, add the digits and then subtract that sum from the original number. Finally, he asks the audience member to read out the digits of this new number in any order omitting one digit. If the digits read out are 1, 3, and 4, the Zoltag announces the missing number. What is it?

Exercise 9. MAML 2002. A magic square is an $n \times n$ array filled with the integers 1, 2, ..., n^2 with the property that each row and each column of the array has the same sum. What is the sum of each row and each column in a 10×10 magic square?

Exercise 10. MAML 2002. How many zeros are there at the end of the decimal expansion of 150! ?