

Combinatorics

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Student: I've added these numbers ten times.

Teacher: Good girl.

Student: And here are my ten answers.

Class Discussion

Four basic combinatorics questions. Combinations with repetitions.

Warm-Up

Exercise 1. How many 5-digit numbers with only odd digits exist? How many six digit numbers are there with at least one odd digit?

Exercise 2. There are two mathematics teachers and ten English teachers at a school. They want to form a homework committee out of eight people. No committee can survive without a mathematician. In how many ways can they form a survivable committee?

Exercise 3. License plates contain one letter, then 3 digits, then another letter. What is the maximum number of cars that can be registered?

Exercise 4. Tanya has 20 students, out which she needs to build a team of 6 for HMMT. In how many ways can she do this? In how many ways can she build 2 teams?

Exercise 5. Are there more 7 digit numbers containing the digit one or not containing the digit one?

Exercise 6. Find the sum of all 7-digit numbers you can get by permuting the digits one through seven.

Exercise 7. Tanya has 20 students at her math club. The club needs to elect a president, a vice-president and a secretary. In how many ways can they do it? In how many ways can they do it if it is allowed for the vice-president and the secretary to be the same person?

Competition Practice

Exercise 8. 2005 AMC 10A, Problem 22. Let S be the set of the 2005 smallest positive multiples of 4, and let T be the set of the 2005 smallest positive multiples of 6. How many elements are common to S and T ?

Exercise 9. 2005 AMC 12A, Problem 18. Call a number "prime-looking" if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

Challenge Problems

Exercise 10. Three cannibals and three missionaries must cross a river. Their boat can only hold two people. If the cannibals outnumber the missionaries, on either side of the river, the missionaries are in trouble (I won't describe the results). Each missionary and each cannibal can row the boat. How can all six get across the river? A more difficult question: what if only one missionary and one cannibal know how to row?

Exercise 11. There are 10 sets of 10 coins. You know how much the coins should weigh. You know all the coins in one set of ten are exactly a hundredth of an ounce off, making the entire set of ten coins a tenth of an ounce off. You also know that all the other coins weight the correct amount. You are allowed to use an extremely accurate digital weighing machine only once.

How do you determine which set of 10 coins is faulty?

Exercise 12. A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. For example, after traveling one mile the odometer changed from 000039 to 000050. If the odometer now reads 002005, how many miles has the car actually traveled?