# Game Theory 2

## Tanya Khovanova

## April 14, 2014

## Class Discussion

Symmetric games. P-positions. N-positions.

## Related to previous homework

**Exercise 1.** For the problems from the previous homework, answer the following questions.

- Are their P-positions?
- If there are P-positions, then what are they?
- What is the winning strategy?
- Is it possible to say who wins with the right strategy without knowing the strategy?
- Are some of these games equivalent?

This is the list of two-player games from the previous homework.

- **Limited Subtraction game.** Game: one heap, and the number of objects that can be taken is 1 or 2. Players take turns, and the player who can't move loses. Who wins? Same game, but the number of objects that can be taken is any number between 1 and $n$.

- **Dividing Piles.** There are two piles with candy. One pile has 20, the other has 21 pieces. In one move a player can eat a pile and divide the other pile into two (not necessarily equal) piles. The person without a move loses.

- **Subtract Divisors Game.** We start with the number 60. In one move a player subtracts the number by one of its divisors. The person who gets to zero loses.

- **Tic-Tac-Toe.**

- **15-Sum Game.** There are nine cards with numbers 1,2,3,4,5,6,7,8,9 written on them. On each turn a player takes a card. The first player to get three cards that sum to 15 wins.

- **Forbidden Factors Game.** Two players play a game. On his/her turn a player can write down an integer between 1 and $n$ inclusive. The next player can't write a number that is a factor of an already written number. The person who can't move loses.

## Game theory

**Exercise 2.** Prove that the P-positions in Nim are defined as XOR of all piles.

**Exercise 3. The Wythoff's game.** Explore the Wythoff game. Wythoff's game is a two-player mathematical game of strategy, played with two piles of counters. Players take turns removing counters from one or both piles; in the latter case, the numbers of counters removed from each pile must be equal. The game ends when one person removes the last counter or counters, thus winning.

## Challenge Problems

**Exercise 4. Chomp.** Chomp is a two-player strategy game played on a rectangular chocolate bar made up of smaller square blocks (cells). The players take it in turns to choose one block and "eat it" (remove from the board), together with those that are below it and to its right. The top left block is "poisoned" and the player who eats this loses.

Explore the game of Chomp. Who wins? The formula for the P-positions of Chomp is not known.