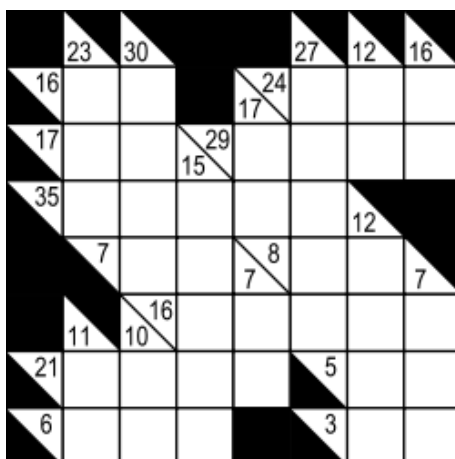# Rotations

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Without geometry life would be pointless.

Warm Up

Exercise 1. A certain sheik named Hassan had eight horses. Four of them were white, three were black, and one was brown. Assuming now that Hassan's horses can talk, how many of them can each say that it is the same color as another one of Hassan's horses?

Exercise 2. If a certain plant were three feet taller, then it would be twice as tall as it would be if it were half a foot less. How tall is the plant?

**Exercise 3.** Solve the following kakuro puzzle.

Rotations

Exercise 4. Prove that a composition of a translation and a rotation is a rotation.

Exercise 5. Segments AB and CD are of equal length and not parallel. Prove that there exists a unique rotation that moves AB to CD, such that point A moves to C and point B moves to D.

Exercise 6. Given three parallel lines a, b and c, build an equilateral triangle ABC with vertices A, B and C belonging to the lines a, b and c correspondingly (see Figure).
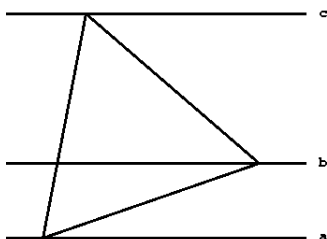
Exercise 7. Find point M inside a triangle ABC such that the sum of the distances from this point to the vertices of the triangle is minimal.

Comeptition Practice

Exercise 8. HMNT 2008. Al has a rectangle of integer side lengths a and b, and area 1000. What is the smallest perimeter it could have?

Exercise 9. HMNT 2008. $ABCDE$ is a regular pentagon inscribed in a circle of radius 1. What is the area of the set of points inside the circle that are farther from A than they are from any other vertex?

Exercise 10. Spivak middle school Olympiad. A rectangle is cut into several rectangles with integer perimeters (in centimeters). Can we conclude that the perimeter of the starting rectangle is an integer (in centimeters)?

Exercise 11. Spivak middle school Olympiad. In an acute triangle the smallest angle is $1/5$ of the largest angle. Find the angles, if all of them are integer degrees and distinct.