Sampling Bias

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May 14, 2012

Class Discussion

Sampling Bias. Polling by calling random phone numbers.

Warm-Up

Exercise 1. An old witch told to her students, "You are too young to look at this. Close your eyes!" All of the boys and one third of the girls closed their right eye. All of the girls and one third of the boys closed their left eye. What percentage of students looked at what they were too young to look at?

Exercise 2. Six baskets have pears, plums, and apples. The number of plums in each basket is equal to the number of apples in the remaining five baskets. The number of apples in each basket equals the number of pears in the remaining five baskets. Prove that the total number of fruits in all the baskets is divisible by 31.

Exercise 3. Alice, Bob, and Carol played a word game: each has to make as many words as possible from the letters of a given word. The scoring is done in the following way: if every one found the same word, the word is crossed out. If two people thought of the same word, the word gets 1 point. If only one person invented a word, the word is given 2 points. Alice got the most number of words and Carol — the least. Is it possible for Carol to get the most number of points, and for Alice — the least?

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Exercise 4. A chess magazine asked their readers to fill in the survey form. Out of all respondents, 90% said that they like math very much. Can we

conclude that 90% of the population likes math? Explain.

Exercise 5. A company wants to have a survey about average health. They stopped random people on the streets and asked them questions. Explain how this type of survey can change the figures.

Exercise 6. A lab director is interested in the percentage of people with HIV. He secretly tested all blood samples that arrived to his lab for HIV. He got 10%. What percentage of the population has HIV?

Competition Practice

Exercise 7. What's more: 200! or 100^{200}?

Exercise 8. You have a piece of chocolate. You are allowed to divided it into three pieces or six pieces. Then you can choose a piece and repeat the procedure again. List all possible number of pieces that you can get.

Exercise 9. Math Festival, Russia, 2012. Bob made a true self-referencing statement, "In this sentence 1/3 of all the digits is 3, and 1/2 of all the digits is 1."

Alex wrote another statement: "In this sentence $1/\#$ of all the digits is $*$, proportions of digits $*$ and $*$ are the same and equal to $1/\#$, and the proportion of the remaining digits is $1/\#$."

Replace asterisks with three distinct digits, and hash symbols with three distinct integers so that the statement is true.

Challenge Problems

Exercise 10. Russian Geometry Olympiad, 2005. It is known that a triangle can be cut into three triangles similar to each other. Prove that it can be cut into any number of similar triangles.