Graph Theory. Basics

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Class Discussion

Graphs, simple graphs, degrees, degree sequences. Königsberg bridges.

Warm-Up

Exercise 1. The number 1000000000030000000000700000000021 is the product of two smaller natural numbers. Find them.

Exercise 2. Bob decided that he will exercise on Wednesdays, Saturdays and also on all odd days of the month. What could be the longest stretch when he has to exercise every day?

Exercise 3. Alice and Bob had 20 AP exams. The number of fives that Alice got is the same as the number of fours Bob got. Similarly, the number of Alice's fours is the same as the number of Bob's threes. The number of Alice's threes is the same as the number of Bob's twos. The number of Alice's twos is the same as the number of Bob's fives. Strangely enough they have the same average grade. How many twos does Alice have?

Exercise 4. The shaded region in the Figure below is bounded by three semi-circles. Cut this region into four congruent parts.

Graph Theory

Exercise 5. Draw all possible simple graphs with 4 vertices.

Exercise 6. What is the largest sum of degrees in a simple graph with n vertices? What is the largest total number of edges in a simple graph with n vertices?
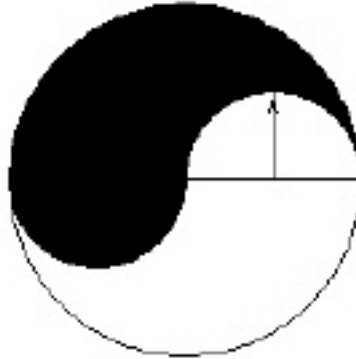
Exercise 7. Draw all possible simple graphs with the degree sequence {2, 2, 2, 1, 1}. Prove that the sum of numbers in any degree sequence of a graph is even. Invent a theorem about the degree sequence and the number of edges.

Competition Practice

Exercise 8. Moscow Olympiad 2012. At a meeting there were n people ($n \geq 1$). It appears that any two people at the meeting shared exactly two common acquaintances.

- Prove that all people have exactly the same number of common acquaintances at this meeting.
- Show that n can be greater than 4.

Challenge Problems

Exercise 9. Here are several dates in Swahili: tarehe tatu Disemba jumamosi; tarehe pili Aprili jumanne; tarehe nne Aprili jumanne; tarehe tano Octoba jumapili; tarehe tano Octoba jumatatu; tarehe tano Octoba jumatano.

The translations in English are given in random order: Monday 5 October; Tuesday 2 April; Wednesday 5 October; Sunday 5 October; Saturday 3 December; Tuesday 4 April.

Write in Swahili: Wednesday 3 April; Sunday 2 December; Monday 1 November.

Exercise 10. All faces of a convex polyhedron are regular pentagons or regular hexagons. How many pentagonal faces does this polyhedron have?