Hanoi Tower

Tanya Khovanova

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Class Discussion

Suppose the tower has n discs. The number of possible positions is 3^n. The number of moves in the optimal strategy is $2^n - 1$. The number of moves in a given position is 3, unless all the discs are on the same posts when it is 2. First move depends on the parity of the number of discs.

Warm-Up

Exercise 1. Out of four statements three are true. Which one is false?

1. Integer N is divisible by 2.
2. Integer N is divisible by 3.
3. Integer N is divisible by 6.
4. Integer N is divisible by 12.

Exercise 2. A cube $3 \times 3 \times 3$ is glued out of 27 cubes $1 \times 1 \times 1$. One drop of glue was used for gluing a pair of neighbouring faces. How many drops total were used?

Exercise 3. Knights always tell the truth. Knaves always lie. Seven knights and knaves set around a round table. They started talking in a clockwise order. The first said to the second, "You are a liar." The second said to the third, "You are a liar." And the third continued calling the fourth one a liar, the fourth called the fifth a liar, the fifth called the sixth a liar, and the sixth called the seventh a liar. What did the seventh person call the first one?

Competition Practice

Exercise 4. Integers 1 through 20 are written on the board. After one number is erased, the arithmetic mean of the leftover numbers is one of the numbers on the board. Find all possible numbers that could have been erased.

Exercise 5. Numbers are placed on the vertices of a 100-gon in such a way that each is an arithmetic mean of its neighbours. Prove that all the numbers are equal.

Exercise 6. A first grader Dan was playing with cards representing digits. He created a correct equality. His baby brother switched two cards and the result became: $314159 + 291828 = 585787$. Which digits were switched?

Exercise 7. Maria was converting all country capitals into numbers by replacing A with 1, B with 2, and so on. She discovered that there is a capital that needed only digits 1 and 2 for its representation: 11122112. What is the capital?

Exercise 8. HMNT 2012. Guts Round. Yuhang is making a bracelet for his one true love using beads. How many distinct bracelets can be made from 2 red beads, 2 green beads, and 2 blue beads, if two bracelets are distinct if and only if one cannot be made into the other through rotations and reflections?

Exercise 9. MAML 2012. Allen, Boris, and Clyde are playing a game. To start the game, each of them is randomly assigned a real number in the interval $[0, 100]$. These are their initial scores. What is the probability that the sum of all three of their initial scores is in the interval $[50, 250]$?

Challenge Problems

Exercise 10. You are sitting at the equator and you have three planes. You would like to fly around the equator. Each plane is full of gas and each has enough gas to take you half way around. Planes can transfer gas between themselves mid-air. You have friends, so that you can fly more than one plane at once. How do you fly around the world?