Fibonacci Numbers

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How to prove that all odd numbers are prime?

Physicist: 3 is prime, 5 is prime, 7 is prime, 9 is an experimental error...

Quantum Physicist: All numbers are equally prime and non-prime until observed.

Computer Scientist: 10 is prime, 11 is prime, 101 is prime...

Programmer: 3 is prime, 5 is prime, 7 is prime, 9 will be fixed in the next release, ...

Windows programmer: 3 is prime. Wait...

Computer programmer: 3 is prime, 5 is prime, 7 is prime, 7 is prime, 7 is prime, 7 is prime, 7 is ...

Computational linguist: 3 is an odd prime, 5 is an odd prime, 7 is an odd prime, 9 is a very odd prime, ...

Philosopher: Why don't we just call all the odd numbers prime and call all the prime numbers odd, that way all the odd numbers would be prime.

Statistician: 100% of the sample 5, 13, 37, 41 and 53 is prime, so all odd numbers must be prime.

Class Discussion

Fibonacci numbers. Lucas numbers. Formula for Fibonacci numbers.

Warm-Up

Exercise 1. What are the indices of even Fibonacci numbers? What are the indices of the Fibonacci numbers divisible by 3?

Exercise 2. Don't read this sentence.

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Exercise 3. Can you continue the Fibonacci sequence to the negative indices?

Exercise 4. Prove that $\sum_{i=0}^{n} F_i = F_{n+2} - 1$.

Exercise 5. Prove that $F_{n+1}^2 = F_n F_{n+2} + (-1)^n$.

Competition Practice

Exercise 6.
- Prove that $n^3 + 5n$ is divisible by 6, for any integer n.
- Prove that $1 + 3 + 5 + ... + (2n - 1) = n^2$.
- Prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$.

Exercise 7. How many binary words of length 11 are there such that every digit appears only an odd number of times in a row?

Exercise 8. Find all prime numbers p such that $p + 10$ and $p + 14$ are also prime.

Challenge Problems

Exercise 9. Mr. House would like to visit his old friend Mr. Street, who is living in the main street of a small village. The main steet has 50 houses divided into two blocks and numbered from 1 to 20 and 21 to 50. Since Mr. House has forgotten the number and likes playing games, he asks a passer-by three questions:

- In which block is it?
- Is the number even?
- Is it a square?

After Mr. House has received the answers, he says: "I'm still doubting, but if you'll tell me whether the digit 4 is in the number, I will know the answer!" Then Mr. House runs to the building in which he thinks his friend is living. He rings, a man opens the door and it turns out that he's wrong. The man starts laughing and tells Mr. House: "Your advisor is the biggest liar of the whole village. He never speaks the truth!" Mr. House thinks for a moment and says "Thanks, now I know the real address of Mr. Street." What is the address of Mr. Street?