Hat Puzzles

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March 10, 2014

— My dear, you love math more than me!
— Of course not, how could you think such a thing!
— Prove it!
— Let A be the set of the objects I love...

Class Discussion

I puzzle with 3 red hat and 2 blue hats. A puzzle with many red and blue hats and people in line.

Warm-Up

Exercise 1. How many 3-digit numbers are divisible by 7?

Exercise 2. Tanya's math club has 17 boys, and 23 of the students program in Java. If 14 of the students that program in Java are girls, then how many boys do not program in Java?

Exercise 3. What is the units digit of the sum: $1! + 2! + 3! + 4! + \ldots + 100!$?

Exercise 4. How many 4-digit numbers have exactly one zero?

Exercise 5. How many 5-digit palindromes are there?

Competition Practice

Exercise 6. 2006 AMC 12B, Problem 7. Mr. and Mrs. Lopez have two children. When they get into their family car, two people sit in the front,

and the other two sit in the back. Either Mr. Lopez or Mrs. Lopez must sit in the driver's seat. How many seating arrangements are possible?

Exercise 7. 2006 AMC 12B, Problem 9. How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?

Exercise 8. 2006 AMC 12B, Problem 18. An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?

Exercise 9. 2006 AMC 12A, Problem 25. How many non-empty subsets S of $\{1, 2, 3, \ldots, 15\}$ have the following two properties?

1. No two consecutive integers belong to S
2. If S contains k elements, then S contains no number less than k.

Exercise 10. 2004 AIME An integer is called *snakelike* if its decimal representation $a_1 a_2 a_3 \cdots a_k$ satisfies $a_i < a_{i+1}$ if i is odd and $a_i > a_{i+1}$ if i is even. How many snakelike integers between 1000 and 9999 have four distinct digits?

Challenge Problems

Exercise 11. Tanya's secretary needs to send out 5 invitation letters for Tanya's math party. In a hurry she messed up the letters and the envelopes. Every letter was mailed to a wrong person. In how many ways could the secretary have messed up?

Exercise 12. A group of hikers went for a hike in a park. At the first bench half of them and one half of one hiker stopped for rest and decided not to continue. At the second bench one half of the remaining group and one half of a hiker stopped for a snack and decided not to continue. At the last bench a similar thing happened. After three benches only one hiker was continuing the trip. How many hikers started the trip?

Exercise 13. How many subsets of the set $\{1, 2, 3, \ldots, 11\}$ can you form in such a way that the subsets do not contain any two consecutive numbers?